**B2.1-B2.2 Tutorial Questions – DFS & Z-Transform and Discrete Systems analysis**

**Q1:** Determine the Discrete Fourier Series coefficients of the following periodic discrete functions using (i) the Discrete Fourier Transform (DFT) method and (ii) using Euler’s relation. Plot the line spectra (Magnitude, Phase and Power) and verify Parseval’s relation for the power in both signals.



**Q2:** Using the DFS formula (DFT) determine the Discrete Fourier Series coefficients for the periodic sequence h[n] with fundamental period N=4, defined as: h[n]=[ 0 1 2 3]

* 1. Make a sketch of the magnitude, power and phase of the line spectrum of h[n] and verify Parseval’s relation for discrete periodic signals for the discrete signal h[n].
  2. What percentage of total power resides in the 1st harmonic of h[n] ? (note: ensure you use k=1 and k=-1 = -1Mod 4 = 3 DFS coefficients when computing the power in the 1st harmonic)

**Q3:** A 50Hz sine waveform is sampled at four times the Nyquist sampling frequency. During the acquisition process the amplitude of the sine wave is clipped to ±0.7071 resulting in a distorted sine wave.

1. Write down the values of one period of the acquired and sampled clipped sinusoid.
2. Make a plot of the underlying sinusoidal signal and the distorted sinusoid on the same plot
3. Using the DFS of the discrete clipped signal, determine the Total Harmonic Distortion (THD) i.e. the % power in the TOTAL harmonics (not including the 1st harmonic) compared to that in the fundamental harmonic (the 1st harmonic). THD is an important aspect in audio, commnications and power systems and should normally be as low as possible. Sometime ‘music’ use THD for artistic reasons.

**Q4**: Find the z-transform of the following signals

(i) x[n]=(0.5)nu[n-2]

(ii) (0.5)nu[n]\*(0.3)nu[n] where \* is the convolution summation

(iii) x[n]=[0,1,2,0,**1**,2,0..] (iv) (0.5)nsin(0.3n)u[n]

**Q5**: Find the inverse Z- transform of the following transforms.

(i) [z4+z 3 +z2+z]/[(z+0.1)(z-0.3)2] \* hint use the shift property

(ii) (1-z-1)4  \* hint expand and write down the answer

(iii) (z4+2z2+3+2z-2+z-4) \*hint: using the fact that IZT(z X(z)) = x[n+1]

**Q6**: Determine the system function H(z), the pole-zero diagrams (for (i) and (iii) only) , Impulse Response, and Frequency Response of the systems described by the following difference equations

(i) y[n]-0.5y[n-1]=2x[n-1]

(ii) y[n]=x[n]-x[n-2]+x[n-4]-x[n-6] : Do not attempt to get the Pole Zero Diagram of this question

(iii) y[n]-0.25y[n-1]-0.4y[n-2]=-x[n]+2x[n-1]

**Q7**: For the following impulse responses of digital filters determine the system function H(z), a linear difference equation that describes the system and an expression for the frequency response for the systems:

(i) h[n]=3(0.25)nu[n-1]

(ii) h[n]=δ[n]-δ[n-5]

(iii) h[n]=2(0.6)nu[n-1]+(0.25)n[cos(πn/6)-2sin(πn/6)]u[n]

**Q8:** Determine the System Function H(z), Pole-Zero Diagram, unit impulse response, unit step response, frequency response, the Zero State Response [yzsr[n]] due to the inputs x1[n] and x2[n] illustrated below, the Zero Input Response [yzir[n]], the steady state response [yss[n]] and the transient response [ytr[n]] of the systems that are described by the following difference equations with given initial conditions:

(i)y[n]=0.5y[n-1]+x[n] with y(-1)=0.25

(ii) y[n]=x[n-2]-x[n-3]-1.5y[n-1]-1.5y[n-2]-0.5y[n-3] with y(-1)=y(-2)=y(-3)=1

Input 1: x1[n]=u[n]-u[n-7]

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0 1 2 3 4 5 6

n

Input 2: x2[n]= 4 +exp(jπn/4)+cos(πn/2)

[Note: This is the Steady state output, so use the fact that if a LTI system with system function H(z) / frequency reponse H(Ω) has an input x[n]=exp(j Ω1 n ) then the steady state output is yss[n]=[ exp(j Ω1 n )] H(Ω)|Ω=Ω1 i.e. The input multiplied by a complex number so only the Magnitude and Phase of the input is altered by the LTI digital filter. ]